Math 70 B Chapter 14

Lesson 54

Section 14.1 <u>only</u>!

Two topics

1) Sequences
2) Series

Day 1: Sequences

Day 2: Series

Day 3: Review sequences + series.

Main goal: Recognize, understand and use

- subscript notation for sequences
- sigma (summation) notation for series

Math 70 Sequences

Objectives: 1) Write (numerical) terms of a sequence given its (algebraic) general term

2) Find an (algebraic) general term given several (numerical) terms of the sequence.

A <u>sequence</u> is a list of numbers which

- follows a pattern
- never ends
- are separated by commas
- is usually called $\{a_n\}$, "the sequence a"

n is a variable with two meanings

1) we actually substitute the value of n into the general (algebraic) term
2) we use n for the subscripted name of that term.

(1) Example: $\{a_n\}$ where $a_n = 2n-7$

$n=3$ gives $a_3 = 2(3)-7$

$$\boxed{a_3 = -1}$$

The third term in the sequence is -1.

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(2) Write the first five terms of a sequence whose general term is $a_n = n^2 - 1$.

step 1: substitute $n=1$ to find a_1

$a_1 = 1^2 - 1 = 1 - 1 = 0$ $\qquad a_1 = 0$

step 2: substitute $n=2$ to find a_2.

$a_2 = 2^2 - 1 = 4 - 1 = 3$ $\qquad a_2 = 3$

step 3: substitute $n=3$ to find a_3.

$a_3 = 3^2 - 1 = 9 - 1 = 8$ $\qquad a_3 = 8$

step 4: substitute $n=4$ to find a_4.

$a_4 = 4^2 - 1 = 16 - 1 = 15$ $\qquad a_4 = 15$

step 5: substitute $n=5$ to find a_5

$a_5 = 5^2 - 1 = 25 - 1 = 24$ $\qquad a_5 = 24$

step 6: write this finite sequence as a list separated by commas:

$\boxed{0, 3, 8, 15, 24}$

These terms of a sequence can be generated using the LIST feature of your GC.

Rev 8-8-13

Name ______________________

Date ______________________

TI-84+ GC 36 Finding Values of a Sequence Using Lists

Objective: Use the sequence operation from 2nd function LIST to calculate terms in a sequence

LIST will do many things, but we're using only the sequence operation, called **seq(** in this exercise.
The **seq(** operation requires four inputs:

1. expression to be evaluated,
2. the index variable used in that expression,
3. the starting value of the index variable, and
4. the ending value of the index variable.

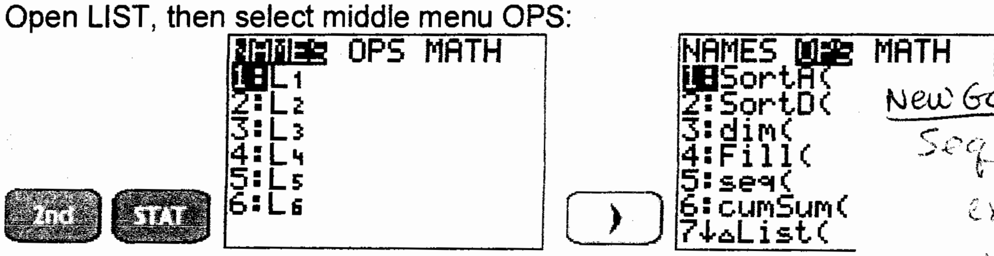
These four are listed one after the other, in order, separated by commas.

Example 1: What does **seq(**1/n, n, 1, 4) **> frac** do?

Answer: It displays the sequence $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right\}$.

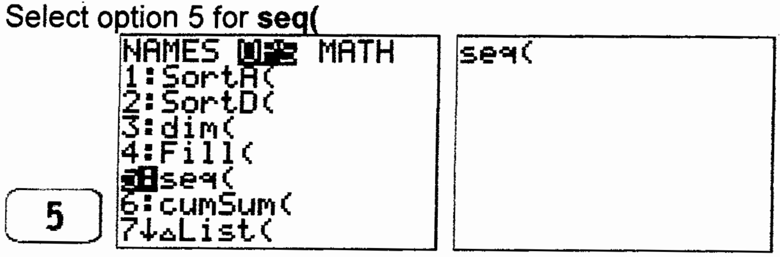
Example 2: Calculate the first 10 terms of a sequence whose general term is $a_n = \frac{(-1)^n}{3n}$ and write the results as reduced fractions (exact answers).

Open LIST, then select middle menu OPS:



New GC's:
seq(
expression:
variable:
start:
end:
step: 1 (always)
Paste ← select, press ENTER

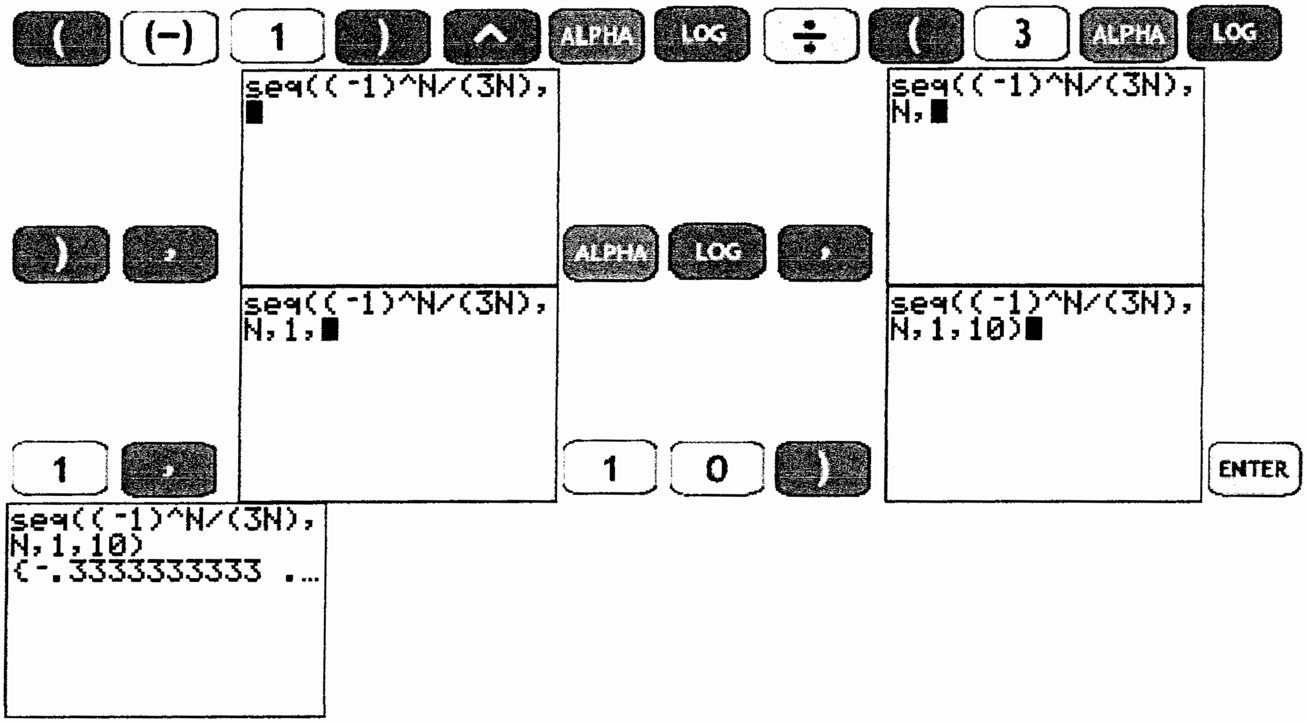
Select option 5 for **seq(**



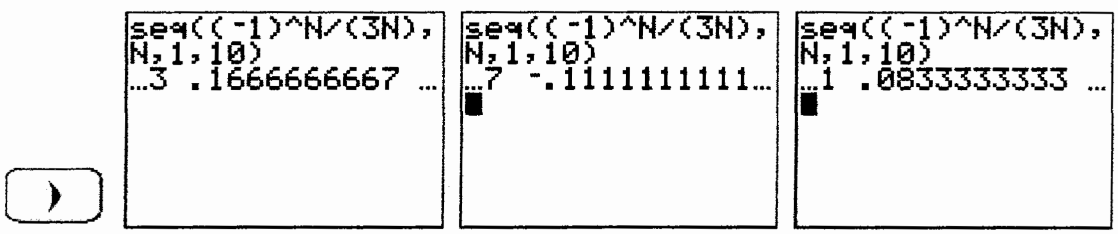
Type the four inputs: (keystrokes and images on next page)

1. Type the algebraic expression for the general term, $a_n = \frac{(-1)^n}{3n}$, followed by a comma.
2. Type the index variable N and a comma.
3. Type the starting value of the index variable, 1, and a comma.
4. Type the ending value of the index variable, 10, and close the parentheses. Press ENTER.

The result is a list of decimals separated by spaces. Use the right arrow to see the entire list.

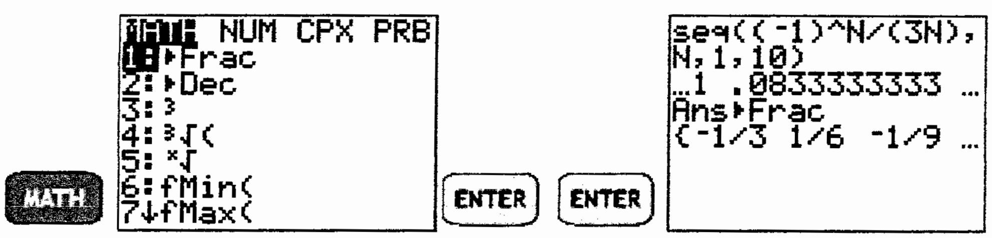


Using the right arrow shows lengthy decimals, which are pretty annoying:



etc.

Convert decimal to fractions as usual: MATH menu, option 1, **>Frac**.



Again, use the right arrow to move through the list.

Answer:

$$\left\{-\frac{1}{3},\frac{1}{6},-\frac{1}{9},\frac{1}{12},-\frac{1}{15},\frac{1}{18},-\frac{1}{21},\frac{1}{24},-\frac{1}{27},\frac{1}{30}\right\}$$

Math 70 Word problems for Sequences

1) The amount of weight, in pounds, a puppy gains in each month of its first year is modeled by a sequence whose general term is $a_n = n + 4$, where n is the number of months since the puppy's birth.

   a. Write the first five terms of the sequence.

   $n=1 \quad a_1 = 1+4 = 5$
   $n=2 \quad a_2 = 2+4 = 6$
   $n=3 \quad a_3 = 3+4 = 7$
   $n=4 \quad a_4 = 4+4 = 8$
   $n=5 \quad a_5 = 5+4 = 9$

   $\boxed{5, 6, 7, 8, 9}$

   b. How much weight should the puppy gain in its 5th month?

   a_5 = amount puppy should gain in 5th month.
   = $\boxed{9 \text{ pounds}}$

   (Big dog!)

2) Mrs. Laser agrees to give Mark an allowance of \$0.10 on the first day of a 14-day vacation, \$0.20 on the second, \$0.40 on the third, etc.

   a. Write the general term of this sequence.

   $$\left.\begin{aligned} a_1 &= .10 = (.10)(1) = (.10)(2^0) \\ a_2 &= .20 = (.10)(2) = (.10)(2^1) \\ a_3 &= .40 = (.10)(4) = (.10)(2^2) \\ a_4 &= .80 = (.10)(8) = (.10)(2^3) \end{aligned}\right\} \quad \boxed{a_n = (.10)(2)^{n-1}}$$

   b. Find the allowance Mark will receive on the last day of vacation.

   Find $a_{14} = (.10)(2^{14-1})$
   $= (.10)(2^{13})$
   $= (.10)(8192)$
   $= \boxed{\$819.20}$

   Either Mark is a very clever negotiator or Mark's mother is very generous!

3) The number of cases of an infectious disease is doubling every year so that the number of cases is $a_n = 75(2)^{n-1}$ where n is the number of the year just beginning.

   a. Find how many cases there will be at the beginning of the 6th year.

   $a_6 = 75(2)^{6-1}$
   $= 75(2)^5$
   $= 75(32)$
   $= \boxed{2400 \text{ cases}}$

   exponent before multiply!

   b. How many cases were there at the beginning of the first year?

   $a_1 = 75(2)^{1-1}$
   $= 75(2)^0$
   $= 75(1)$
   $= \boxed{75 \text{ cases}}$

(3) Find the general term a_n for the sequence
$-2, 1, 4, 7, 10, \ldots$

step 1: Identify the first term and what is done to get each term.

$a_1 = -2$

$a_2 = 1 \quad \leftarrow \quad -2+3=1 \quad$ so $\quad a_2 = a_1+3$

$a_3 = 4 \quad \leftarrow \quad 1+3=4 \quad\quad a_3 = a_2+3$

$a_4 = 7 \quad \leftarrow \quad 4+3=7 \quad\quad a_4 = a_3+3$

$a_5 = 10 \quad \leftarrow \quad 7+3=10 \quad\quad a_5 = a_4+3$

step 2. Using a_1 in place of the first term, write every term as an expression using a_1 only. (not a_2, a_3, etc.)

$a_1 = -2$

$a_2 = 1 = -2+3 = a_1+3$

$a_3 = 4 = 1+3 = a_2+3 = (a_1+3)+3 = a_1+2(3)$

$a_4 = 7 = 4+3 = a_3+3 = a_1+2(3)+3 = a_1+3(3)$

$a_5 = 10 = 7+3 = a_4+3 = a_1+3(3)+3 = a_1+4(3)$

step 3: *Notice* Repeated addition becomes adding a multiplied term.

step 4: Identify the index of each term with results of step 2. Write the result using n

$n=1 \quad a_1 = -2$

$n=2 \quad a_2 = a_1+3$

$n=3 \quad a_3 = a_1+2(3)$

$n=4 \quad a_4 = a_1+3(3)$

$n=5 \quad a_5 = a_1+4(3)$

notice a counting pattern and extend it to all terms.

$n=1 \quad a_1 = a_1+0(3) \quad$ $n=1$ but calculation uses $0=(n-1)$

$n=2 \quad a_2 = a_1+1(3) \quad$ $n=2$ but calculation uses $1=(n-1)$

$n=3 \quad a_3 = a_1+2(3) \quad$ $n=3$ but calculation uses $2=(n-1)$

So $a_n = a_1+(n-1)\cdot 3 = \boxed{-2+3(n-1)}$ or $-2+3n-3 = \boxed{-1+3n}$

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When finding the general term of a sequence, there are four basic strategies

I Add the same number each time

II Multiply the same number each time

III Add a predictable, but increasing number each time

IV Reciprocal of one of the above.

V Alternating signs – special version of II.

Strategy I: Arithmetic Sequence $a_n = a_1 + d(n-1)$

(3) $-2, 1, 4, 7, 10, \dots$

step 1: Notice that each term is 3 more than the term before it.

3 is called the common difference; d is its variable

step 2: The general term is $a_n = a_1 + d(n-1)$

The first term plus a multiple of d.

$a_1 = -2$

$d = 3$

$a_n = -2 + 3(n-1)$

step 3: simplify by distribute and combine like terms

$a_n = -2 + 3n - 3$

$\boxed{a_n = 3n - 5}$

step 4: check

$n=1 \quad 3(1)-5 = -2$ ✓

$n=2 \quad 3(2)-5 = 1$ ✓

$n=3 \quad 3(3)-5 = 4$ ✓

$n=4 \quad 3(4)-5 = 7$ ✓

$n=5 \quad 3(5)-5 = 10$ ✓

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(4) Find the general term of the sequence
$-2, -6, -18, -54$

step 1: Identify the first term and what is done to get each term

$a_1 = -2$

$a_2 = -6 = -2+4$ or -2×3

$a_3 = -18 = -6-12$ or -6×3

$a_4 = -54 = -18-36$ or -18×3

we don't use add because it's not the same every time

we use multiply because it's multiply by 3 each time

step 2: Using a_1 in place of the first term, write every term as an expression using a_1 only (not a_2, a_3, etc.)

$a_1 = -2$

$a_2 = -6 = a_1\cdot 3$

$a_3 = -18 = a_2\cdot 3 = (a_1\cdot 3)\cdot 3 = a_1\cdot 3^2$

$a_4 = -54 = a_3\cdot 3 = (a_1\cdot 3^2)\cdot 3 = a_1\cdot 3^3$

$a_5 = -162 = a_4\cdot 3 = (a_1\cdot 3^3)\cdot 3 = a_1\cdot 3^4$

step 3: * Notice * Repeated multiplication becomes multiplying by a factor with an exponent.

step 4: Identify the index of each term with the results from step 2.

$n=1$ $a_1 = -2 \longrightarrow a_1\cdot 3^0$ $n=1$ but calculation uses $n-1=0$

$n=2$ $a_2 = a_1\cdot 3 \longrightarrow a_1\cdot 3^1$ $n=2$ but calculation uses $n-1=1$

$n=3$ $a_3 = a_1\cdot 3^2$ $n=3$ but calculation uses $n-1=2$

$n=4$ $a_4 = a_1\cdot 3^3$ $n=4$ but calculation uses $n-1=3$

so $a_n = a_1\cdot(3)^{n-1}$

$a_n = -2\cdot(3)^{n-1}$ or $\boxed{-2(3)^{n-1} = a_n}$

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Strategy II: Geometric Sequence $a_n = a_1 \cdot r^{n-1}$

(4) $-2, -6, -18, -54, \ldots$

Step 1: Notice that each term is 3 times the previous term.

3 is called the common ratio, r is its variable

Step 2: The general term is $a_n = a_1 \cdot r^{n-1}$

The first term times a power of r.

$a_1 = -2$

$r = 3$

$a_n = -2(3)^{n-1}$

Step 3: If the coefficient (a_1) has a common factor r, you can combine + simplify.

Step 4: check

$n=1 \quad -2(3)^{1-1} = -2\cdot 3^0 = -2\cdot 1 = -2$ ✓

$n=2 \quad -2(3)^{2-1} = -2\cdot 3^1 = -2\cdot 3 = -6$ ✓

$n=3 \quad -2(3)^{3-1} = -2\cdot 3^2 = -2\cdot 9 = -18$ ✓

$n=4 \quad -2(3)^{4-1} = -2\cdot 3^3 = -2\cdot 27 = -54$ ✓

$\boxed{a_n = -2(3)^{n-1}}$

or

$\boxed{a_n = -2\cdot 3^{n-1}}$

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Strategy III: Squared Sequence $a_n = c_1 n^2 + c_2 n + c_3$

(5) 5, 11, 21, 35, 53, ...

$+4 \quad +10 \quad +14 \quad +18$

Step 1: Notice that we add an increasing number each time.

Step 2: Substitute the first three terms to get a linear system with three unknown variables, c_1, c_2, and c_3. using $a_n = c_1 n^2 + c_2 n + c_3$

$n=1$: $a_1 = c_1 \cdot 1^2 + c_2 \cdot 1 + c_3$

$5 = c_1 + c_2 + c_3$ ← 1st equation

$n=2$ $a_2 = c_1 \cdot 2^2 + c_2 \cdot 2 + c_3$

$11 = 4c_1 + 2c_2 + c_3$ ← 2nd equation

$n=3$ $a_3 = c_1 \cdot 3^2 + c_2 \cdot 3 + c_3$

$21 = 9c_1 + 3c_2 + c_3$ ← 3rd eq'n

Step 3: Solve linear system using matrices in your GC!

$$\begin{bmatrix} 1 & 1 & 1 & 5 \\ 4 & 2 & 1 & 11 \\ 9 & 3 & 1 & 21 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$c_1 = 2$
$c_2 = 0$
$c_3 = 3$

Step 4: Substitute back + simplify

$a_n = 2n^2 + 0n + 3$

$\boxed{a_n = 2n^2 + 3}$

Step 4: check.

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Strategy IV. Reciprocal of a previous pattern

⑥ $\frac{1}{6}, \frac{1}{9}, \frac{1}{12}, \frac{1}{15}, \ldots$

step 1: notice that the denominators are an "add 3" pattern. Follow strategy 1.

$$\begin{cases} a_1 = 6 \\ d = 3 \end{cases}$$

$a_n = 6 + 3(n-1)$ arithmetic sequence

$a_n = 6 + 3n - 3$

$a_n = 3n + 3$

step 2: Take reciprocal of result.

$$\boxed{a_n = \frac{1}{3n+3}}$$

step 4: check

Practice

⑦ $6, 3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \ldots$

step 1: Notice we divide by 2 = multiply by $\frac{1}{2}$ each step. This is strategy II.

step 2: $a_1 = 6$

$r = 1/2$

$$\boxed{a_n = 6\left(\frac{1}{2}\right)^{n-1}}$$

step 3 optional. $a_n = 3 \cdot 2^1 \cdot \frac{1^{n-1}}{2^{n-1}}$

$$\boxed{a_n = 3 \cdot \frac{1}{2^{n-2}}} \quad \text{or} \quad \boxed{a_n = \frac{3}{2^{n-2}}}$$

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(8) $\frac{3}{8}, \frac{7}{8}, \frac{11}{8}, \frac{15}{8}, \frac{19}{8}, \ldots$

Step 1: Notice that we add $\frac{4}{8} = \frac{1}{2}$ each time.

This is strategy I.

Step 2: $a_1 = \frac{3}{8}$

$d = \frac{1}{2}$

$a_n = \frac{3}{8} + \frac{1}{2}(n-1)$

Step 3: $a_n = \frac{3}{8} + \frac{1}{2}n - \frac{1}{2}$

$$\boxed{a_n = \frac{1}{2}n + \frac{1}{8}}$$

Math 70 Extra

Find the general term a_n of a sequence given the first several terms of the sequence.

Hints

Look for counting numbers

(3) $1, 2, 3, 4, 5 \Rightarrow a_n = n$

Look for reciprocals

(4) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5} \Rightarrow a_n = \frac{1}{n}$

Look for multiples

(5) $2, 4, 6, 8, 10 \Rightarrow a_n = 2n$

(6) $3, 6, 9, 12, 15 \Rightarrow a_n = 3n$

(7) $-4, -8, -12, -16, -20 \Rightarrow a_n = -4n$

(8) $\frac{1}{5}, \frac{1}{10}, \frac{1}{15}, \frac{1}{20}, \frac{1}{25} \Rightarrow a_n = \frac{1}{5n}$

Look for powers

(9) $2, 4, 8, 16, 32 \Rightarrow a_n = 2^n$

(10) $3, 9, 27, 81, 243 \Rightarrow a_n = 3^n$

(11) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32} \Rightarrow a_n = \frac{1}{2^n}$

Special power is alternating signs:

(12) $-1, +1, -1, +1, -1 \Rightarrow a_n = (-1)^n$

Look for squares

(13) $1, 4, 9, 16, 25 \Rightarrow a_n = n^2$

Look for offsets

(14) $0, 1, 2, 3, 4 \Rightarrow a_n = n-1$

(15) $6, 9, 12, 15, 18 \Rightarrow a_n = 3(n+1)$

(16) $1, 2, 4, 8, 16 \Rightarrow a_n = 2^{n-1}$

(17) $3, 6, 11, 18, 27 \Rightarrow a_n = n^2+2$

Then look for combinations of these!

Math 70 Extra

Find the indicated term for each sequence whose general term is given.

(18) $a_n = \dfrac{n-4}{(-2)^n}$; a_6

substitute $n=6$ into the general term

$a_6 = \dfrac{6-4}{(-2)^6} = \dfrac{2}{64} = \boxed{\dfrac{1}{32}}$

(19) $a_n = 8 - n^2$; a_{20}

$n=20$ $\quad a_{20} = 8 - 20^2 = \boxed{-392}$

(20) $a_n = \dfrac{(-1)^n}{2n}$; a_{100}

$n=100$ $\quad a_{100} = \dfrac{(-1)^{100}}{2(100)} = \boxed{\dfrac{1}{200}}$

* Use even/odd powers!

(21) $a_n = \dfrac{n+3}{n+4}$; a_8

$n=8$ $\quad a_8 = \dfrac{8+3}{8+4} = \boxed{\dfrac{11}{12}}$

(22) $a_n = 5^{n+1}$; a_3

$n=3$ $\quad a_3 = 5^{3+1} = 5^4 = \boxed{625}$

(23) $a_n = \dfrac{n}{n+4}$; a_{24}

$n=24$ $\quad a_{24} = \dfrac{24}{24+4} = \dfrac{24}{28} = \boxed{\dfrac{6}{7}}$

(24) $a_n = 100 - 7n$; a_{50}

$n=50$ $\quad a_{50} = 100 - 7(50) = \boxed{-250}$

(25) $a_n = -n^2$; a_{15}

$n=15$ $\quad a_{15} = -(15)^2 = \boxed{-225}$

Math 70 Extra Practice

To practice writing numerical terms, cover the right side of the page and use the general term to find the first four terms.

To practice finding general terms, cover the left side of the page and use the four terms on the right to find a general term.

(26) $a_n = -7n+2$

(27) $a_n = -4(2)^{n+1}$ or $-16(2^{n-1})$ or -2^{n+3}

(28) $a_n = -3n^2-1$

(29) $a_n = 9n-1$

(30) $a_n = 7n+3$

(31) $a_n = -4n-5$

(32) $a_n = -6n+9$

(33) $a_n = 3\left(\frac{1}{2}\right)^n$

(34) $a_n = 3\left(\frac{2}{5}\right)^n$

(35) $a_n = 5\left(\frac{2}{3}\right)^{n-1}$

(36) $a_n = n^2$

(37) $a_n = n^2-3$

(38) $a_n = n^2+4$

(39) $a_n = 2n^2$

(26) $-5, -12, -19, -26$

(27) $-16, -32, -64, -128$

(28) $-3, -12, -27, -48$

(29) $8, 17, 26, 35$

(30) $10, 17, 24, 31$

(31) $-9, -13, -17, -21$

(32) $3, -3, -9, -15$

(33) $\frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}$

(34) $\frac{6}{5}, \frac{12}{25}, \frac{24}{125}, \frac{48}{625}, \frac{96}{3125}$

(35) $5, \frac{10}{3}, \frac{20}{9}, \frac{40}{27}$

(36) $1, 4, 9, 16$

(37) $-2, 1, 6, 13$

(38) $5, 8, 13, 20$

(39) $2, 8, 18, 32$

M40

(26) Find the general term of the sequence $-5, -12, -19, -26 \ldots$

step 1 $a_1 = -5$ $= a_1$ $= a_1 - 0(7)$

$a_2 = -12 = -5 - 7$ $= a_1 - 7$ $= a_1 - 1(7)$

$a_3 = -19 = -12 - 7$ $= a_2 - 7 = (a_1 - 7) - 7 = a_1 - 2(7)$

$a_4 = -26 = -19 - 7$ $= a_3 - 7 = (a_1 - 2(7)) - 7 = a_1 - 3(7)$

step 2

step 3: Repeated subtraction becomes subtracting a multiplied term.

step 4: $n=1$ $a_1 = a_1 - 0(7)$ $n=1$ but calc uses $n-1 = 1-1 = 0$.

$n=2$ $a_2 = a_1 - 1(7)$ $n=2$ $n-1 = 2-1 = 1$

$n=3$ $a_3 = a_1 - 2(7)$ $n=3$ $n-1 = 3-1 = 2$

$n=4$ $a_4 = a_1 - 3(7)$ $n=4$ $n-1 = 4-1 = 3$.

$$a_n = -5 - (n-1)\cdot 7$$
$$= \boxed{-5 - 7(n-1)}$$
$$= -5 - 7n + 7$$
$$= \boxed{2 - 7n}$$

(27) Find the general term of the sequence $-16, -32, -64, -128$

step 1: $a_1 = -16$ $= a_1 \cdot 2^0$

$a_2 = -32 = -16 - 16$ or -16×2 $= a_1 \cdot 2^1$

$a_3 = -64 = -32 - 32$ or -32×2 $= a_1 \cdot 2^2$

$a_4 = -128 = \underbrace{-64 - 64}_{\text{The \# subtracted changes}}$ or $\underbrace{-64 \times 2}_{\text{The \# multiplied stays the same}}$ $= a_1 \cdot 2^3$

step 2 ↑

step 3: Repeated multiplication is multiplication by a factor with an exponent.

step 4: $n=1$ $a_1 = a_1 \cdot 2^0$ $n=1$ but calc uses $n-1 = 1-1 = 0$.

$n=2$ $a_2 = a_1 \cdot 2^1$ each time, $(n-1)$.

$n=3$ $a_3 = a_1 \cdot 2^2$

$n=4$ $a_4 = a_1 \cdot 2^3$

$$a_n = a_1 \cdot 2^{n-1}$$

$$\boxed{a_n = -16(2^{n-1})}$$

But $16 = 2\cdot 2\cdot 2\cdot 2 = 2^4$. This general term can be written also as

$$a_n = \underbrace{-(2^4)(2^{n-1})}_{\text{exp laws}} = -2^{n-1+4} = \boxed{-2^{n+3} = a_n}$$

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(34) Find the general term of the sequence $\frac{6}{5}, \frac{12}{25}, \frac{24}{125}, \frac{48}{625}, \frac{96}{3125}$

step 1: $a_1 = \frac{6}{5}$

$a_2 = \frac{12}{25} = \frac{6}{5} - \frac{18}{25}$ or $\frac{6 \times 2}{5 \times 5}$

$a_3 = \frac{24}{125} = \frac{12}{25} - \frac{36}{125}$ or $\frac{12 \times 2}{25 \times 5}$

$a_4 = \frac{48}{625} = \underbrace{\frac{24}{125} - \frac{72}{625}}_{\text{changes}}$ or $\underbrace{\frac{24 \times 2}{25 \times 5}}_{\text{always } \times \frac{2}{5}}$

– NO –   – YES –

step 2: $a_1 = \frac{6}{5} \qquad = a_1 \times \left(\frac{2}{5}\right)^0$ $\qquad n = 1$ but calc uses $n-1=0$

$a_2 = a_1 \times \frac{2}{5} = a_1 \times \left(\frac{2}{5}\right)^1$ $\qquad n=2 \qquad n-1=1$

$a_3 = a_2 \times \frac{2}{5} = a_1 \times \frac{2}{5} \times \frac{2}{5} = a_1 \times \left(\frac{2}{5}\right)^2$ $\qquad n=3 \qquad n-1=2$

$a_4 = a_3 \times \frac{2}{5} = a_2 \times \left(\frac{2}{5}\right)^2 \times \frac{2}{5} = a_1 \times \left(\frac{2}{5}\right)^3$ $\qquad n=4 \qquad n-1=3$

step 3: Repeated multiplication = multiplication by a term w/ exponent.

$a_n = a_1 \times \left(\frac{2}{5}\right)^{n-1}$

$$\boxed{a_n = \frac{6}{5}\left(\frac{2}{5}\right)^{n-1}}$$

notice $6 = 2 \times 3$ $5 = 5 \times 1$ so $\frac{6}{5} = \left(\frac{2}{5}\right) \cdot 3$

This general term can also be written as:

$$a_n = 3 \cdot \underbrace{\left(\frac{2}{5}\right)^1 \cdot \left(\frac{2}{5}\right)^{n-1}}_{\text{exp laws — add exp.}}$$

$$\boxed{a_n = 3\left(\frac{2}{5}\right)^n}$$